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**Group Assignment using AMPL**

the farmer problem

**Problem statement**

The following report is a summary that is based on a group assignment using an optimization model for a given problem and implementing it in the program AMPL. The given problem is called, in this case, the **farmer problem**.

First of all, it needs to be stressed out that a farmer has 500 hectars(ha) of land where he could raise the following agricultural products: **wheat, corn, sugar beet and potato**. In this case the farmer has a constraint that he can either grow sugar or potato and not both but at least one of the two. He knows that at least 200 tons (T) of wheat and 240 (T) of corn are needed for cattle feed. These amounts can be raised on the farm or bought from a wholesaler. Any production that exceeds the feeding requirement would be sold. Selling prices amount to 170 euros and 150 euros per ton of wheat and corn respectively.

A profitable crop is sugar beet, which sells at price €/T. However, the European Commission imposes a quota on sugar beet production and any amount that is higher than the quota can be sold only at a price less than €/T. The farmer’s quota for next year equates to 6000T. Based on his previous experience, the farmer knows that the mean yield depends on both season and weather and so does the price.

A profitable crop is potato, which also sells at a price of €/T. However, the European Commission also imposes a quota on potato production. Any amount higher than the quota can be sold only at price lower than price €/T. The farmer’s quota for next year amounts to 6000T. The crops potato and sugar beets are complementary to each other and only 1 is grown in one season. During the winter time potato is grown and in summer sugar beet is grown.

The maintenance price for each crop is also mentioned taking all other factors into consideration. Maintenance cost also depends on both season and weather conditions which will further be explained in detail in the following paragraphs.

When it comes to tax, it needs to be stressed out that, in this case, it is also seen as a constraint for the farmer and he needs to pay a fixed amount of tax based on hectares of the land. That will be also explained in detail.

The main objective is to know how much land the farmer needs to devote to each crop in order to be able to minimize costs.

**The explanation of variables.**

The table below will show the structure and the current prices of the agricultural products shown below. First, it is essential to mention that the farmer is planning to grow agricultural products like wheat, corn, sugar beet and potato. The total surface of land available equates to 500 hectares and the following table will analyze, on the one hand, the prices and, on the other hand, the different costs for these products

To explain each separately **Yield** means the amount of crop that would be produced from each hectare of land.

**Planting Cost**: refers to the cost required to plant the crop per hectare.

**Selling Price:** refers to the selling price of the crops per ton.

**Purchase Price:** refers to purchase price of the crop per ton.

**Minimum Requirement:** refers to the minimum T of crops that needs to be produced.

**Maintenance Price**: refers to the cost for electricity, water etc. for each hectare of land.

**Tax Price:** The tax paid in euros per hectare per crop

**Winter Bad Weather:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Wheat** | **Corn** | **Sugar Beet** | **Potato** |
| **Yield (T/Ha)** | 3.5 | 4 | 10 | 12 |
| **Planting cost (e/Ha)** | 250 | 330 | 360 | 360 |
| **Selling price (e/T)** | 170 | 150 | 36 ( < 6,000 T) 10 ( > 6,000 T) | 36 ( < 6,000 T)  10 ( > 6,000 T) |
| **Purchase price (e/T)** | 238 | 210 | NA | NA |
| **Minimum requirement (T)** | 200 | 240 | NA | NA |
| **Maintenance Price** | 60 | 65 | 85 | 75 |
| **Tax Price** | 1.8 | 2.1 | 3.2 | 2.2 |
| **Total available land: 500 Ha** | | | | |

**Results:**

The weather condition “winter bad” is considered as showing extreme temperatures and snow for which the farmer assigned the following values concerning the area of land:

\* 365 hectars for wheat

\* 60 hectars for corn

\* 75 hectars for potato

This brings the farmer to a cost saving that equates to 42,180.50 euros.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Area of Land (x) hec. | 365 | 60 | 0 | 75 |
| Tons of crops to be purchased | 0 | 0 | - | - |
| Minimization value(euros) | -42180.5 | | | |
| Winter Bad | | | | |

**Winter Good Weather:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Yield (T/Ha) | 1.5 | 2 | 8 | 10 |
| Planting cost (e/Ha) | 100 | 130 | 160 | 160 |
| Selling price (e/T) | 170 | 150 | 36 ( < 6,000 T) 10 ( > 6,000 T) | 36 ( < 6,000 T)  10 ( > 6,000 T) |
| Purchase price (e/T) | 238 | 210 | NA | NA |
| Minimum requirement (T) | 200 | 240 | NA | NA |
| Maintenance Price | 60 | 65 | 85 | 75 |
| Tax Price | 1.8 | 2.1 | 3.2 | 2.2 |
| Total available land: 500 Ha | | | | |

**Results:**

If the farmer plans to farm in the winter season and the weather conditions are considered being “good” – meaning the winter season has just started, the area of land in hectares the farmer wants to assign for wheat amounts to 134, for corn 120 and for potato 246 hectars. The tons of crops to be purchased are 0 again and, in this case, there is loss of 9927.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Area of Land (x) hec. | 134 | 120 | 0 | 246 |
| Tons of crops to be purchased | 0 | 0 | - | - |
| Minimization value(euros) | 9926.666667 | | | |
| Winter Good | | | | |

**Winter Normal:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Yield (T/Ha) | 2.5 | 3 | 6 | 8 |
| Planting cost (e/Ha) | 150 | 230 | 260 | 260 |
| Selling price (e/T) | 170 | 150 | 36 ( < 6,000 T) 10 ( > 6,000 T) | 36 ( < 6,000 T)  10 ( > 6,000 T) |
| Purchase price (e/T) | 238 | 210 | NA | NA |
| Minimum requirement (T) | 200 | 240 | NA | NA |
| Maintenance Price | 60 | 65 | 85 | 75 |
| Tax Price | 1.8 | 2.1 | 3.2 | 2.2 |
| Total available land: 500 Ha | | | | |

**Results:**

If the farmer expects a winter considered being normal to grow agricultural products, the area of land will be assigned to everything except sugar beet. The area of land assigned to wheat amounts to 345

hectares, to corn 80 hectares and 75 hectares for potatoes. If the tons of crops that need to be purchased is 0 again then the farmer will experience a cost saving 15,137.50 euros.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Area of Land (x) hec. | 345 | 80 | 0 | 75 |
| Tons of crops to be purchased | 0 | 0 | - | - |
| Minimization value(euros) | -15137.5 | | | |
| Winter Normal | | | | |

**Summer Good Weather:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Yield (T/Ha) | 3.5 | 4 | 20 | 20 |
| Planting cost (e/Ha) | 150 | 230 | 250 | 260 |
| Selling price (e/T) | 175 | 155 | 36 ( < 6,000 T) 10 ( > 6,000 T) | 36 ( < 6,000 T)  10 ( > 6,000 T) |
| Purchase price (e/T) | 242 | 215 | NA | NA |
| Minimum requirement (T) | 200 | 240 | NA | NA |
| Maintenance Price | 55 | 65 | 80 | 70 |
| Tax Price | 1.8 | 2.1 | 3.2 | 2.2 |
| Total available land: 500 Ha | | | | |

**Results:**

In the following chart the area of land in hectares, the tons of crops to be purchased and the minimization value in euros are displayed for different agricultural products and for different seasons. The seasons were divided in summer good, summer bad, summer normal and winter good, winter bad and winter normal. If we, first of all, want to calculate the minimization value for the season summer good, then it is essential to say that the area of land that is available for wheat equates to 365 hectares, for corn 60 hectares, for sugar beet 75 and for potatoes it is zero. If the tons of crops that the farmer wants to purchase equates to 0 for both wheat and corn, then we get a value minimization of 118548euros.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Area of Land (x) hec. | 365 | 60 | 75 | 0 |
| Tons of crops to be purchased | 0 | 0 | - | - |
| Minimization value(euros) | -118548 | | | |
| Summer Good | | | | |

**Summer Bad:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Yield (T/Ha) | 1.5 | 2 | 20 | 20 |
| Planting cost (e/Ha) | 150 | 230 | 260 | 260 |
| Selling price (e/T) | 170 | 150 | 36 ( < 6,000 T) 10 ( > 6,000 T) | 36 ( < 6,000 T)  10 ( > 6,000 T) |
| Purchase price (e/T) | 238 | 210 | NA | NA |
| Minimum requirement (T) | 200 | 240 | NA | NA |
| Maintenance Price | 50 | 60 | 75 | 65 |
| Tax Price | 1.8 | 2.1 | 3.2 | 2.2 |
| Total available land: 500 Ha | | | | |

**Results:**

If we apply the same principle when the weather changes from good to bad in the summertime, then the value minimization equates to 27900 euros. That is achieved with the following management: the area of land available for growing wheat is 134 hectares, for corn 66 hectares, for sugar beet 300 hectares and for potatoes 0 hectares again. The tons of crops that the farmer plans to purchase are 0 for wheat and 107 for corn.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Area of Land (x) hec. | 134 | 66 | 300 | 0 |
| Tons of crops to be purchased | 0 | 107 | - | - |
| Minimization value(euros) | -27900 | | | |
| Summer Bad | | | | |

**Summer Normal:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Yield (T/Ha) | 2.5 | 3 | 20 | 20 |
| Planting cost (e/Ha) | 150 | 230 | 260 | 260 |
| Selling price (e/T) | 170 | 150 | 36 ( < 6,000 T) 10 ( > 6,000 T) | 36 ( < 6,000 T)  10 ( > 6,000 T) |
| Purchase price (e/T) | 238 | 210 | NA | NA |
| Minimum requirement (T) | 200 | 240 | NA | NA |
| Maintenance Price | 50 | 60 | 75 | 65 |
| Tax Price | 1.8 | 2.1 | 3.2 | 2.2 |
| Total available land: 500 Ha | | | | |

**Results:**

If we further look at the minimization of values when the weather conditions change from good to normal, then the saving of costs amounts to -65920euros. That is possible by assigning 120 hectares of land for growing wheat, 80 hectares for growing corn and 300 hectares for sugar beet. The tons of crops that the farmer wants to purchase when the weather in summer is normal amounts to 0 for both wheat and corn.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Wheat | Corn | Sugar Beet | Potato |
| Area of Land (x) hec. | 120 | 80 | 300 | 0 |
| Tons of crops to be purchased | 0 | 0 | - | - |
| Minimization value(euros) | -65920 | | | |
| Summer Normal | | | | |

**Mathematical Formulas:**

**Decision Variables:**

**x1, x2, x3, x4** = Ha of land devoted to wheat, corn, sugar beets and potato respectively

**w1, w2** = tons of wheat and corn to be sold

**y1, y2** = tons of wheat and corn to be purchased

**w3** = tons of sugar beets sold at the favorable price (≤6,000 T)

**w4** = tons of sugar beets sold at the lower price (≥6,000T)

**w5** = tons of sugar beets potato at the favorable price (≤6,000 T)

**w6** = tons of sugar beets potato at the lower price (≥6,000T)

**u = 1** means **sugar beet** is Grownand **v = 1** means **potato;** please note **u + v =1**

meaning both can’t be together.

**Optimization Model:**

minimize Total cost:

sum {i in CROPS} cplant[i] \* x[i]

+ sum {i in CROPS1} (pbuy[i] \* y[i]- psale[i] \* w[i])

+ sum{i in CROPS} (mprice[i]\*x[i])

+ sum{i in CROPS1} (tax[i]\*y[i])+ sum{i in CROPS2}(u)\*(tax[i]\*(wbelow[i]+wabove[i]))+ sum{i in CROPS3}(v)\*(tax[i]\*(wzbelow[i]+wzabove[i]))

- sum {i in CROPS2} (u)\*(pvbelow \* wbelow[i] + pvabove \*wabove[i])

- sum {i in CROPS3} (v)\*(potbelow \* wzbelow[i] + potabove \*wzabove[i]);

**What Does the Code mean?**

The above optimization model gives the difference of total money spent on crops and the total money earned by selling the part of these crops. Thus, the formula breaks down into –

* Cost of plantation of the crops
* Purchase cost of the crops
* Maintenance cost for different crops
* Taxes incurred on selling different crops

**Constraints:**

subject to [Total land] ≤ 500 which implies he can either utilize his full or choose to leave some amount of land

[Ensure min wheat production] ≥ 200

[Ensure min corn production] ≥ 240

[Production limit on sugar] ≤ 6, 000

[Production limit on potato] ≤ 5, 000

X1, x2, x3, x4, y1, y2 w1, w2, w3, w4, w5, w6 ≥ 0

Min Land to be utilized for sugar / potato should be more than 75 and is also seen as a constraint as the farmer needs one of these crops for daily consumption.